R-Zi

IGEE-UMBB (Spring 2017)

EE 174 : Recitations set 1

1. Given the sets A, B show that $\overline{A \cup B} = \overline{A} \cap \overline{B}$

2. Consider the set $\mathcal{R}$ of real numbers

   (a) Is $\{\mathcal{R}, +\}$ a group where $(+)$ is the usual addition ?
   b) Repeat for $\{\mathcal{R}, \times\}$ where $(\times)$ is the usual multiplication ?

3. Let $\{G, *\}$ be an Abelian group, show that for all $a, b, x \in G$ we have:

   a) $a * x = b * x \Rightarrow a = b$
   b) $a * x = b \Rightarrow x = a^{-1} * b$
   c) $(a * b)^{-1} = b^{-1} * a^{-1}$

4. Consider $\{R^2, +, \circ\}$ where the binary operations $(+)$ and $(\circ)$ are defined as follows:
   $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and $(x_1, y_1) \circ (x_2, y_2) = (x_1 x_2, y_1 y_2)$.
   Is $\{R^2, +, \circ\}$ a ring? Is it commutative? Does it have a unit element?

5. Consider $\{R^2, +, \circ\}$ where the binary operations $(+)$ and $(\circ)$ are defined as follows:
   $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and $(x_1, y_1) \circ (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + y_1 x_2)$.
   Is $\{R^2, +, \circ\}$ a field ?

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EE 174 : Recitations set 2

1. Which of the following is a vector space ?
   a) $S = \{(x,y,z,w)/x+y-z+w=0\}$
   b) $S = \{ae^{x}+be^{-x}/a,b \text{ real}\}$ — S ≠ is not a sebset
   c) $S = \{(x,y,z)/x+y+z=1\}$ — (No). $\theta \notin S$ $(0+0+0 \neq 1)$
   d) $S = \{f: R \to R/ \frac{d^2f}{dx^2}+f=0\}$ — S is a sebset

2. Let $v \neq \theta$ be a vector of the vector space $V(\mathcal{F})$; Show that:
   (a) $0v = \theta$
   b) $\alpha v = \theta$ if and only if $\alpha = 0$
   c) $\alpha v = \beta v$ if and only if $\alpha = \beta$

3. Let $V(\mathcal{R})$ be the space of real-valued functions
   a) Is the set U of **even** real-valued functions a subspace ?
   b) Is the set W of **odd** real-valued functions a subspace ?
   c) From the fact that $f(x) = \frac{1}{2}[f(x)+f(-x)] + \frac{1}{2}[f(x)-f(-x)]$, what can conclude on the relationship between $V(\mathcal{R}), U$ and W ?

4. Consider again $S = \{(x,y,z)/x+y+z=1\}$
   a) Is S a suspace of $\mathcal{R}^3$? $\theta \notin S \Rightarrow$ No .
   b) Suppose we define on S the following binary operations:
   $(x_1,y_1,z_1)+(x_2,y_2,z_2) = (x_1+x_2-1, y_1+y_2, z_1+z_2)$ and
   $\alpha(x,y,z) = (\alpha x-\alpha+1, \alpha y, \alpha z)$; Is S a vector space ?
   c) Comment your result.

5. Let S, T be subspaces of $V(\mathcal{F})$
   a) Are $\bar{S}$ and $\bar{T}$ subspaces of $V(\mathcal{F})$?
   b) Show that a necessary condition for $S \cup T$ to be a subspace is that one is contained in the other.

   Hint: Use contradiction to show that if the condition is not satisfied, $S \cup T$ is not closed under addition.

$R\rightleftharpoons 2^1$

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EE 174 : Recitations set 3

1. Check the linear independence of the following vectors of R^3
   a) $\{(0,0,-1);(1,0,4)\}$
   b) $\{(1,1,0);(-1,2,0)\}$
   c) $\{(1,1,0);(-1,1,2);(1,-1,0)\}$

2. Let P_2 be the space of real polynomials of degree≤ 2 ; Check the linear independence of the following sets
   a) $\{1,x,x^2\}$
   b) $\{-1+4x-x^2, 1+7x, 2+3x+x^2\}$
   c) Can we select a basis for P_2 from these sets?

3. Let $S = \{v_1, v_2, \cdots, v_k\}$ be a subset of the vector space $V(F)$; Show that $[S] = [v_1, v_2, \cdots, v_k] = [v_1, v_2, \cdots, v_k, v_{k+1}]$ **if and only if** $v_{k+1} \in [v_1, v_2, \cdots, v_k]$ for any $v_{k+1} \in V(F)$. What conclusions can you draw from this result?

4. Let S, T be linearly independent subsets of vectors of the vector space $V(F)$
   a) Is $S \cap T$ linearly independent ? Justify.
   b) Are the complements $\overline{S}$ and $\overline{T}$ linearly independent? Justify.

5. Let P_3 be the space of real polynomials of degree≤ 3 ; Is the subset $\{1, 1+x, 1+x+x^2, 1+x+x^2+x^3\}$ a basis for P_3? If so, express $p(x) = x^2 + x^3$ in this basis.

6. Find a basis and the dimension of the solution space of the following system of linear equations:

$$x_1 - 4x_2 + 3x_3 - x_4 = 0$$
$$2x_1 - 8x_2 + 6x_3 - 2x_4 = 0$$

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EE 174 : **Recitations set 4**

1. Let W be a subspace of dimension m of the vector space V of dimension n ; Show that:
   (a) $m \leq n$
   (b) if $m = n$ then $W = V$

2. Consider $T : R^2 \to R^2$ defined by $T(x,y) = (x+y, x+y)$
   (a) Is T linear ?
   (b) Is $v_1 = (1,0)$ and $v_2 = (1,1)$ a basis for R^2? Are their images also a basis for R^2?What can you conclude?

3. Consider $L : R^3 \to R^2$ defined by $T(x,y,z) = (x,y)$
   (a) Is L linear? What does it represent geometrically?
   (b) Is $v_1 = (1,0,0)$ and $v_2 = (0,1,0)$ a basis for R^3? Are their images a basis for the codomain? What do you conclude?

4. Let $L : V \to W$ be a linear mapping ; Show that:
   a) L is **onto** if and only if $r(L) = \dim W$
   b) If $\dim V = \dim W$, then L is **one-to-one** if and only if L is **onto**.

5. Find the nullity of the linear mapping $L : P_n \to R$ defined by $L[p(x)] = \int_0^1 p(x)dx$ where $p(x) = a_0 + a_1 x + \cdots + a_n x^n$; Deduce the rank of L

6. Consider the linear transformation $T : R^3 \to R^3$ defined by $T(x,y,z) = (x+y+z, y-z, x-y+z)$; Is T nonsingular ? If so, determine $T^{-1}(x,y,z)$.

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EE 174 : Recitations set 5

1. Let the linear transformation $T : R^3 \to R^3$ be defined by $T(x,y,z) = (x+2y-z, y+z, x+y-2z)$; Find the dimension and a basis for the range space $\mathcal{R}(T)$ and the null space $\mathcal{N}(T)$.

2. Consider $T : R^2 \to R^2$ defined by $T(1,1) = (2,0)$ and $T(1,0) = (-1,0)$
   (a) Find the matrix representation A relative to the basis $\{(1,1);(1,0)\}$
   (b) Find the matrix representation B relative to the standard basis
   (c) Find the matrix representation C relative to the basis $\{(1,-1);(1,1)\}$

3. Consider the space of real functions $V = \{a\cos x + b\sin x / a,b \in R\}$ and $D : V \to V$ defined by $D[f(x)] = \frac{df(x)}{dx}$;
   Is $\{\cos x, \sin x\}$a basis for V? If so, give the matrix representation A of D relative to this basis.

4. Perform the following operations, if defined:

i) $\begin{pmatrix} 5 & -1 & 2 \\ 6 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 5 & -1 & 2 \\ 6 & 1 & 1 \end{pmatrix}$ ii) $4\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} + 5\begin{pmatrix} -1 & 4 \\ -2 & 1 \end{pmatrix}$

iii) $3\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} + 2\begin{pmatrix} 1 & 1 & 4 \\ 3 & 0 & 5 \end{pmatrix}$ iv) $\begin{pmatrix} 1 & 1 & -1 \\ 4 & 0 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 & -1 \\ 3 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix}$

v) $\begin{pmatrix} 2 & -7 \\ 7 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 & 5 \\ -1 & 1 & 1 \\ 3 & 8 & 4 \end{pmatrix}$

5. Given $A = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 2 \\ 4 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} -2 & 3 & 0 \\ -4 & 1 & 1 \end{pmatrix}$ compute when defined AB, BA, ABC, CBA and comment.